On the cap-set problem and the slice rank polynomial method

In 2016, Ellenberg and Gijswijt made a breakthrough on the so-called cap-set problem, which is an important problem in additive combinatorics. The problem asks the following: what is the maximum size of a subset of $\mathbb{F}_p^n$ without a three-term arithmetic progression (i.e. without three distinct vectors $x$, $y$ and $z$ in the set such that $x+z=2y$)? Ellenberg and Gijswijt proved that any such subset has size at most $2.756^n$. Their proof was later reformulated by Tao, introducing what is now called the slice rank polynomial method. This method has also led to progress on several other problems in extremal and additive combinatorics.

This talk will give some background on the cap-set problem, and then explain Tao's proof of the Ellenberg-Gijswijt bound via the slice rank polynomial method. Sauermann will also discuss some related problems and results.

Lisa Sauermann is an Assistant Professor in Mathematics at the Massachusetts Institute of Technology (MIT). Her main research area is extremal combinatorics, and in her work she often applies algebraic and probabilistic methods. She is originally from Dresden and completed her undergraduate studies in Bonn. She obtained her PhD in 2019 at Stanford University, and afterwards spent time as a postdoctoral fellow at Stanford University and the Institute for Advanced Study (IAS) in Princeton before moving to MIT in 2021.