
An Introduction to Approximation Theory

1. Introduction and Preliminary Observation
 - Norms, Convexity, Strict Convexity, Uniform Convexity
2. Weierstrass Theorem and Bernstein Polynomial
 - Weierstrass Theorem and the Bernstein Constructive proof of convergence
3. Best Uniform Approximation
 - Sufficient Conditions for Uniqueness of the Best Approximation, Characterization of the Best Approximation in the Uniform Norm, Jackson Theorems and It's Applications
4. Interpolation and Optimal Approximation
 - Optimal interpolation and Chebyshev polynomials
5. Best Approximation in L_2 -norm and Orthogonal Polynomials
 - Orthogonal Polynomials and Best L_2 Approximation, Trigonometric Approximation
6. Properties of orthogonal polynomials
 - Orthogonal polynomials and it's applications in best L_2 approximation
7. Best L_1 -norm Approximation
 - Existence and Uniqueness of the best approximation in L_1 norm
8. Piecewise Polynomials and Spline Theory
 - Interpolation and approximation using splines
9. High Dimensional Interpolation and Approximation using Radial Basis Functions
 - Radial basis functions and interpolation in higher dimension
10. Compressed Sensing and Best k -term Approximation
 - Introduction to CS and Best k -term approximation

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- and some recent papers.