BMS Summer School 2014[1]

Applied Analysis for Materials

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Abstracts of Contributed Talks

Laura Bittner

Optimal Reliability in Design for Fatigue Life under Cyclic Thermomechanic Loading

Blades in gas turbines are exposed to very strong forces caused by a.o. rotation, air pressure and temperature. As they have a large impact on stress and strain states, they influence the reliability of the component. The deterministic LCF-lifetime (Low Cycle Fatigue) can be determined using the PDE of thermal elasticity and a material theoretic model. But, since the appearance of cracks at the surface is not predictable, a stochastic approach will be integrated. The resulting cost functional $J(\Omega, T(\Omega), u(\Omega))$ determines the probability of failure depending on temperature $T(\Omega)$, displacement $u(\Omega)$ and material behavior and shall be minimized with regard to the component's shape Ω.

Paul Cazeaux

Coarse multiscale timestepping for problems in plasma physics with equation-free projective integration.

Multiscale plasma problems are hard to simulate because the physics of micro and macro scales are strongly linked. We propose a coarse-grained numerical scheme, based on equation-free projective integration, for a plasma system modelled by the Vlasov-Poisson equations. In this work, a kinetic particle-in-cell (PIC) code is used to simulate the micro scale dynamics. As a first test case, we simulate the propagation and steepening of a nonlinear ion acoustic wave and show good agreement with the full PIC simulation. The speedup of the projective integration scheme over the PIC scheme scales linearly with the system size.

Marko Erceg

One-scale H-measures

Microlocal defect functionals (H-measures, H-distributions, semiclassical measures etc.) are objects which determine, in some sense, the lack of strong compactness for weakly convergent L^p sequences. In contrast to the semiclassical measures, H-measures are not suitable to treat problems with a characteristic length (e.g. thickness of a plate). Luc Tartar overcame the

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mentioned restriction by introducing 1-scale H-measures, a generalisation of H-measures with a characteristic length. Moreover, these objects are also an extension of semiclassical measures, being functionals on continuous functions on a compactication of $\mathbb{R}^d\setminus\{0\}$. We improve and generalise Tartar's localisation principle for 1-scale H-measures from which we are able to derive known localisation principles for H-measures and semiclassical measures. The localisation principle for H-measures has already been successfully applied in many fields (compactness by compensation, small amplitude homogenisation, velocity averaging, averaged control etc.), and the new results expected to have an even wider class of possible applications. This is a joint work with Nenad Antonić and Martin Lazar.

Avtandil Gachechiladze

An Alternative Monotonicity Method in Quasi-Variational Inequalities

We consider the variational inequalities with unilateral and bilateral obstacles for second order bilinear elliptic form. The domain is bounded and the obstacles may appear in domain and on the boundary as well. We give some monotone dependence results between the solutions and the data of the variational inequalities. It gives an opportunity to construct the monotonicity method for quasi-variational inequalities when the obstacle operator is not monotone in L_2. As an example of the application of these results we consider *implicit Signorini problem (ISP)*, the quasi-variational inequality with the unilateral implicit obstacle operator on the boundary, which is not monotone in L_2 . Using the mentioned monotonicity results we show the unique solvability of the problem and construct the iteration schemes for the solution. Several statements can be considered for the *ISP* in elliptic case: *ISP* with double boundary obstacles, also *ISP* with the obstacles in domain. Due to the mentioned monotonicity properties of the solutions of variational inequalities, the similar results can be obtained as for the classical statement of the *ISP*.

Some of the monotonicity results can be generalized for the evolutionary variational inequalities. Considering *ISP* as the parabolic quasi-variational inequality we obtain the similar results as in the elliptic case.

Héctor Andrade Loarca

Homogenization of an acoustic metafluid using two scale convergence

An acoustic metafluid is a fluid with different properties to those found in nature (it can be a perfect reflectant or absorbent of mechanical waves), this is caused by the microscopic arrangement of their components (whose characteristic length is smaller than the wavelength of the incident mechanical wave). In this talk I want to explain a model of a microscopic periodic arrange of two different fluids and a method of homogenization called Two-scale convergence, which show that this model of fluid have metamaterial properties. Also, I want to show some simulations of these fluids that I made.

Kristina Martin

Optimal Control in a Free Boundary Fluid-Elasticity Interaction

The problem we consider is controlling turbulence inside fluid flow in the case of a free boundary fluid-elasticity interaction. We prove existence of optimal control acting on the body of the fluid and we derive the necessary optimality conditions.

William Minvielle
(extended talk to the lectures of Claude Le Bris)

Two topics in stochastic homogenization

This presentation is divided in two parts.

In the first part, we introduce a variance reduction approach, based on a control variate technique, for the homogenization of a random, linear elliptic second order partial differential equation set on a bounded domain in $\mathbb{R}^d$. The random diffusion coefficient matrix field $A(\frac{x}{\epsilon}, \omega)$ is assumed to be uniformly elliptic, bounded and stationary ("periodic in law"). In the limit when $\epsilon \to 0$, the solution of the equation converges to that of a homogenized problem of the same form, the coefficient field of which is a deterministic and constant matrix A^* given by an average involving the so-called corrector function that solves a random auxilliary problem set on the *entire* space.

In practice, the corrector problem is approximated on a bounded domain Q_N as large as possible. A by-product of this truncation procedure is that the *deterministic* matrix A^* is approximated by a *random*, apparent homogenized matrix $A_N(\omega)$. We therefore introduce a variance reduction approach to obtain practical approximations of A^* with a smaller variance in order to reduce the statistical error. This is a joint work with F. Legoll (École des Ponts, INRIA).

In the second part, we focus on an inverse problem in stochastic homogenization. The setting is slightly different since it involves discrete partial differential equations. The goal is to recover the two parameters of the law of the random variable $A(0, \omega)$. Using only a few macroscopic quantities, we proceed with a Newton algorithm on a least square formulation to show the identification is possible. Additionally, the error committed is controlled by the statistical error introduced in the homogenization procedure (forward problem). This is a joint work with F. Legoll (École des Ponts, INRIA), M. Simon (ENS Lyon UMPA), A. Obliger (PECSA, Univ. Paris VI).

Nachiketa Mishra

A comparative study on iterative solvers for FFT-based homogenization of periodic media

We are concerned with an FFT-based numerical homogenization method proposed by Moulinec and Suquet in 1994 for problems characterized by high-resolution microstructural scans. Recently [J. Vondřejc, J. Zeman & I. Marek (2013), arXiv:1311.0089], we have interpreted the method as a Galerkin scheme generating a system of linear equations for an unknown

$x \in \mathbb{E} \subset \mathbb{R}^n$ in the form

$$GAx = GAE \text{ with } G = F^{-1}\widehat{G}F. \tag{1}$$

Here, $G \in \mathbb{R}^{n\times n}$ is an orthogonal projection from $\mathbb{R}^n$ to $\mathbb{E}$, expressed in terms of the inverse and forward Fourier transform matrices F^{-1} and $F \in \mathbb{R}^{n\times n}$ and a block-diagonal matrix $\widehat{G} \in \mathbb{R}^{n\times n}$, block-diagonal $A \in \mathbb{R}^{n\times n}$ stores the problems coefficient, and $b \notin \mathbb{E}$ is a given vector. In the present contribution, we benchmark the performance of several Krylov solvers when applied to (1).

Markus Mittnenzweig

Entropy production inequalities for heat equations

We interpret the heat equation as an equation modelling thermal conductivity in a material. This physical setting naturally provides us with a given concave entropy function that is different from the standard Boltzmann entropy. In the case of an ideal solid it is given by $f(u) = \log(u)$ and we also consider the case $f(u) = u^c$ with $c < 1$. These entropies are used as Lyapunov functions for heat equations and we derive entropy-entropy-production inequalities.

Maia Mrevlishvili

The Neumann Boundary Value Problems of Statics of Thermo-Electro-Magneto Elasticity Theory

We investigate the interior and exterior Neumann boundary value problems for the system of statics of the thermo-electro-magneto elasticity theory. Using the potential method and the theory of integral equations we prove the existence results. We show that the solutions can be represented by the single layer potentials. In the case of an exterior unbounded domain, it is shown that the unknown density of the potential is deïňĄned uniquely by the corresponding system of integral equations, while in the case of an interior domain of ïňĄnite diameter the corresponding system of integral equations are not solvable for arbitrary right hand side data. We establish the necessary and suïňČcient conditions for the system of integral equations (and thus for the interior Neumann boundary value problem) to be solvable. The basis of the null-space of the corresponding adjoint operator is constructed explicitly which gives us possibility to write the necessary and suïňČcient conditions in eïňČcient form.

Andrew Tapay

A model for studying double-exponential growth in the 2D Euler equations

We introduce a model for the 2D Euler equation that is designed to study, for short time, whether double-exponential growth is possible at an interior point of the flow.

Zivorad Tomovski

Hilfer-Prabhakar derivatives and some applications

This talk contains new results recently published in Applied Mathematics and Computation, Vol.242, 576-589 (2014), (http://www.sciencedirect.com/science/article/pii/S0096300314008297) supported by Einstein Visitor Grant and prepared at WIAS Berlin. We present here a generalization of Hilfer derivatives in which Riemann-Liouville integrals are replaced by more general Prabhakar integrals. We analyze and discuss its properties. Furthermore, we show some applications of these generalized Hilfer-Prabhakar derivatives in classical equations of mathematical physics such as the heat and the free electron laser equations, and in difference-differential equations governing the dynamics of generalized renewal stochastic processes.

Göksu Topkarcı

A Fourier pseudo spectral method for the higher order boussinesq equation

In this study I will consider the higher-order Boussinesq (HBQ) equation. I will propose a Fourier pseudo-spectral numerical method for the HBQ equation. Then I prove the convergence of numerical scheme in the energy space. I will consider three test problems concerning the propagation of a single solitary wave, the interaction of two solitary waves and the solutions that blow up in finite time. The numerical results show that the Fourier pseudo-spectral method provides highly accurate results.

Manoj K. Yadav

Domain decomposition method and asynchronous time integration for parabolic PDEs

Evolution of time dependent physical quantities, such as current, heat etc., in composite materials are modelled by initial boundary value problems for parabolic PDEs. These physical quantities follow diffrenent evolution patterns in different parts of the composite material depending on the material properties, size of constituent material subdomains, coupling scheme, etc. Therefore, the stability and accuracy requirements of a numerical integration scheme may necessitate domain dependent time discretization of the problem. Parabolic problems are usually solved by discretizing spatially using finite elemets and then integrating over time using discrete solvers. We propose an asynchronous multi-domain time integration scheme for solving initial boundary value problems of parabolic PDEs. For efficient parallel computing of large problems, we present the dual decomposition method with local Lagrange multipliers to ensure the continuity of the primary unknowns at the interface between subdomains. We also propose a multi-time step coupling method which enables us to use domain dependent Rothe method on different parts of a computational domain and thus provide an efficient and robust approach to solving large scale composite material problems more accurately.