

Master course- University of Potsdam
Prof. Dr. S. Paycha
Winterterm 2012-2013, starting October 15th.

Regularisation methods in mathematics and physics

Mon : 2.15pm-3.45pm, Room 1.19.1.19; **Tues** : 10.15am-11.45 am, Room 1.08.0.59

Regularisation methods play an important role in various fields of mathematics and physics, where they are used to make sense of divergent expressions. They play a central role in number theory where one is typically confronted with divergent sums, and in quantum physics, where one typically needs to evaluate a divergent integral. We shall introduce various analytic tools and methods in relation with regularisation, which are useful basic tools for any mathematician or physicist. Here are some of the topics which will be discussed :

- Extension of homogeneous distributions
- The Gamma function
- The residue of a symbol ; uniqueness
- Regularised integrals on symbols : discrepancies
- Comparing different regularisation methods
- The Euler-Maclaurin formula
- Regularised discrete sums on symbols
- Discrepancies, uniqueness results
- The zeta function

If time allows, we shall discuss extensions of these regularisation methods to traces of operators, with an introduction to pseudodifferential operators on manifolds.

Prerequisites : Calculus I and II, complex analysis. Some knowledge of differential geometry is welcome but not necessary.

References :

- P. Cartier, An introduction to zeta functions, in "From number theory to physics", ed. M. Waldschmidt et al. 1992
- G. Hardy, Divergent series, Oxford University Press, 1967
- J. Collins, Renormalisation, Cambridge University Press, 1984
- L. Hoermander, The Analysis of Linear Partial Differential Operators I : Distribution Theory and Fourier Analysis, Springer Verlag (1990) (2nd edition)
- S. Paycha, Regularised integrals, sums and traces, University Lecture Notes, AMS (to appear)