Calculus of Variations and Regularity Theory

Lecturer: Jan Kristensen
Place: Rudower Chaussee 26, 0'311
Time: Thursdays 1.15-2.45 pm and 3.30-5.00 pm [1]

Synopsis: The calculus of variations is one of the oldest subjects of mathematics, yet it remains very active and is still evolving fast. Besides its mathematical importance and its links to other branches of mathematics, including geometry and partial differential equations, it is widely used in engineering, physics, economy and biology.

This course aims to give an introduction to the modern theory of existence and regularity of minimizers of multiple integrals in the calculus of variations. The emphasis will be on multi-dimensional and vectorial problems. Prior knowledge of elementary functional analysis, Lebesgue and Sobolev spaces is assumed. The last lectures will cover recent results on the partial regularity of minimizers of convex and quasiconvex integrals. Tutorials will be organized on request.

Existence of minimizers: The direct method. Weak and strong convergence. Oscillations and concentrations. Sequential weak lower semicontinuity. Quasiconvexity in the sense of Morrey.

Regularity of minimizers: Linear elliptic equations and systems. Quadratic functionals. Rank-one convexity and Legendre-Hadamard condition. De Giorgi-Nash-Moser theory (sketch only). Difference-quotient method. Higher differentiability. Version of Gehring's Lemma. The direct approach to regularity. Partial regularity of minimizers: Evans' theorem. Hausdorff dimension and porosity of singular set.

Reading List (incl. further reading):

B. Dacorogna. *Direct methods in the calculus of variations*. Springer, 1989.

E. Giusti. *Direct methods in the calculus of variations*. World Scientific, 2003.

J. Kristensen and G. Mingione. The singular set of ω-minima. *Arch. Ration. Mech. Anal.* 177 (2005), 93–114.

J. Kristensen and G. Mingione. The singular set of minima of integral functionals. *Arch. Ration. Mech. Anal.* 180 (2006), 331–398.

J. Kristensen and G. Mingione. The singular set of Lipschitzian minima of multiple integrals. *Arch. Ration. Mech. Anal.* 184 (2007), 341–369.

J. Maly and W.P. Ziemer. *Fine regularity of solutions of elliptic partial differential equations*. Amer. Math. Soc., 1997.

G. Mingione. Regularity of minima: An invitation to the dark side of the calculus of variations. Applications of Math. 51 (2006), 355–425.

S. Müller. *Variational models for microstructure and phase transitions*. MPI Leipzig, Lecture note 2/1998. http://www.mis.mpg.de/preprints/ln/index.html

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[1]First lecture Thursday 21/6, last lecture Thursday 19/7, 2007