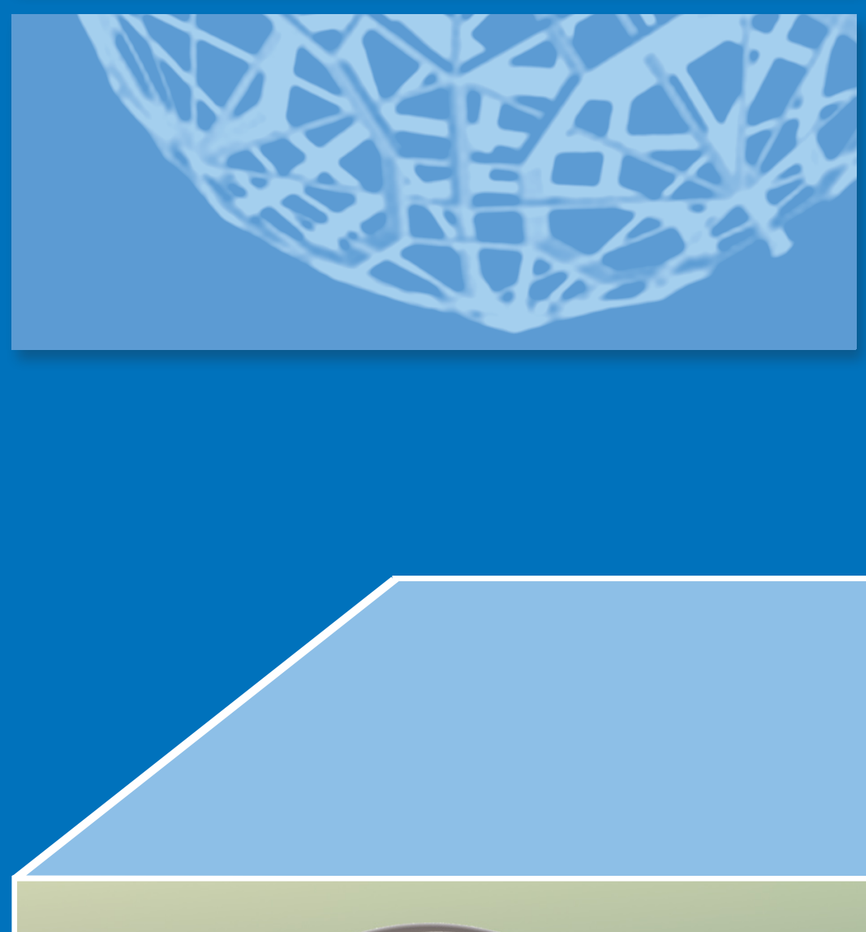
Berlin Mathematical School

BMS Friday Colloquium



Friday 29 April 2011 at 14:15

Tea before the lecture begins at 13:00

BMS Loft, Urania, An der Urania 17, 10787 Berlin

John M. Sullivan

(TU Berlin)

Ropelength Criticality for Knots and Links

What is the shape of a knot tied tight in rope? The ropelength problem asks us to minimize the length of a knot or link in space, subject to a thickness constraint that keeps a unit tube around the curve embedded.

In joint work with Jason Cantarella, Joe Fu and Rob Kusner, we derive a Balance Criterion giving necessary and sufficient conditions for a space curve to be ropelength-critical. Our approach is modeled on rigidity theory for frameworks and uses a new infinite-dimensional version of the Kuhn-Tucker theorem.

In terms of the core curve, the thickness constraint has two parts: an upper bound on curvature and a self-contact condition. The curvature bound is especially difficult to handle, as the curve may fail to be twice differentiable. In the end, we express thickness as the minimum of a compact family of smooth functions in order to apply Clarke's theorem on the derivative of such a minimum.

Using our balance criterion, we can give explicit descriptions of several tight links. The tight configuration of the Borromean rings, for instance, is piecewise smooth with 42 pieces. Even two simply clasped ropes surprising geometric behavior: there is a slight gap between them when they are pulled tight.

John M. Sullivan is Professor of Mathematical Visualization at the Technische Universität Berlin. After earlier degrees from Harvard and Cambridge, he received his Ph.D. in Mathematics from Princeton in 1990. He held professorships at the Universities of Minnesota and Illinois before moving in 2003 to Berlin. Sullivan's research is in the areas of optimal geometry, geometric knot theory, and discrete differential geometry. His mathematical artworks - computer-generated prints, sculptures and videos - have been exhibited around the world.

www.math-berlin.de